



## Evaluation of stress intensity factor for semi elliptical and quarter elliptical crack in pressure vessels using finite element methodology

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### Abstract

The integrity of major components of process equipment industry should be evaluated and maintained during operation. In order to maintain their integrity fracture mechanics evaluation of component is necessary. In pressure vessels cracks may develop during manufacturing and operating condition. For linear elastic fracture mechanics evaluation of pressure vessel, determination of stress intensity factor (SIF) is an important part to predict further behaviour.

In the present study the problem of calculation of stress intensity factors of semi-elliptical and quarter-elliptical crack located in stress concentrated areas of pressure vessel is numerically solved by advanced three dimensional finite element analysis. The crack locations considered for analysis are shell, nozzle to shell junction and shell to head junction. The crack shapes are determined using various non-destructive techniques and guidelines given in ASME Sec VIII Div3 and API-579-1/ASME-FFS-1. The influence of flaw shape on variation of stress intensity factor along the crack front is studied with the help of parametric design concept. Sub modelling method is employed in the analysis for SIF which seems to be very efficient for SIF calculation and reduces the computational time up to great extent. The obtained solutions for both semi-elliptical and quarter-elliptical crack are compared with those in literature and standards given by ASME & API codes.

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### Introduction

In process equipment industry it is widely accepted that existence of crack is unavoidable because of limitations on manufacturing and welding processes [1]. During operation, nucleation of crack may start at stress concentration areas such as shell to head junction and shell to nozzle junction. The application of fracture mechanics in pressure vessel design along with the help of non-destructive inspection is found to be very effective way of dealing with the problem. The evaluation of fracture parameter stress intensity factor (SIF) decides the criticality of crack shape.

Large amount of work has been performed on SIF determination of semi-elliptical crack away from the discontinuity and number of analytical solutions are developed for that.[2] In the present study the finite element method (FEM) is employed to calculate the SIFs of semi-elliptical crack located at stress concentration areas. The analytical method of stress influence coefficient and Newman – Raju solutions are used for validation of FEM solution. These methods are applied on steam drum equipment in which Case I is the simple case of crack away from any discontinuity. Case II is crack in shell nozzle junction and Case III is crack in shell head junction. An ordinary FEM approach for above three cases would be inefficient and extremely time-consuming and therefore the methodology would not be applied. To overcome this difficulty sub-modelling method is to be applied which is based on St. Venat's principle which suggests far away from loading regions and constraint boundaries, mechanical behaviour is not affected. The flow chart of sub-modelling method is outlined in Fig.1.

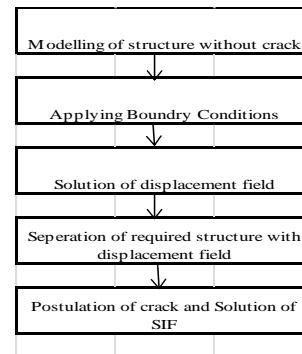


Figure 1: Flow chart illustrating the procedure for sub-modelling

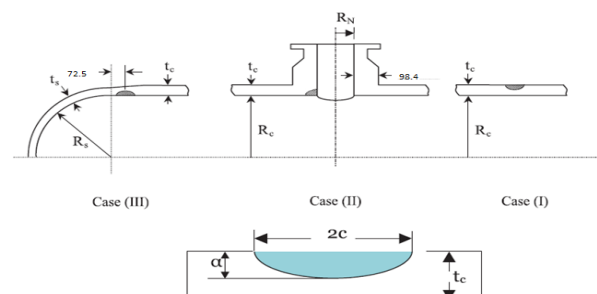


Figure 2: Three cases schematic representation

**Experimental**

**Finite Element Methodology**

Fracture analysis is widely used to predict component failure caused by pre-existing small cracks, allowing one to take precautions to prevent further crack growth or to determine the remaining life of the structure. To obtain the fracture damage, stress intensity factors (SIFs) must be evaluated accurately. Because it is difficult to determine accurate SIFs using a closed-form analytical solution for cracks in complex structures, finite-element analysis is used instead.

**Interaction Integral Method**

Performs the SIF calculation during the solution phase of the analysis and stores the results for later post processing. In J-integral method, J-integral is calculated first. J-integral is the value of a contour integral as shown in Eq

$$J = \int_{\Gamma} \left( W dy - T \cdot \frac{\delta u}{\delta x} ds \right)$$

Where W is the strain energy density. T is the traction vector on a plane defined by the outward normal. u is the displacement vector. C is an arbitrary contour surrounding the crack tip with initial and final points lying on the two crack surfaces, as shown in Fig. 1 and ds is the element of arc along the contour C. J-integral characterizes the energy released associated with the crack growth and is independent of the contour.

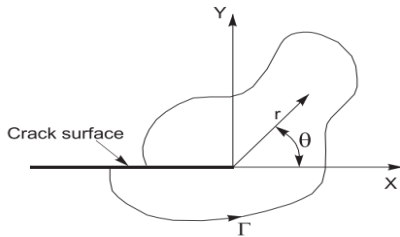


Figure 3: J integral Contour

In linear elastic fracture mechanics, J-integral is proportional to the square of the crack tip stress intensity factor. For plane stress conditions, the stress intensity factor can be calculated by

For Plane Stress:  $K_I = \sqrt{JE}$   
 For Plane Strain:  $K_{I} = \sqrt{JE / (1-\nu^2)}$

**Displacement Extrapolation Method**

The displacement extrapolation method is based on the nodal displacements around the crack tip. To obtain a good representation of the crack-tip quarter-point isoparametric elements are used as suggested by Barsoum and Hensell and Shaw [3]. The  $1/\sqrt{r}$  linear-elastic singularity for stresses and strains is obtained by shifting a quarter to the crack tip the midside nodes of all surrounding elements.

**Stress Influence Coefficient Method**

As per ASME Sec VIII Div3 [4] this method may be used to find stress intensity factor for cracks Type A (Surface Cracks). The same method is also valid for calculation of SIF due to thermal gradients or residual stresses. It may be used to calculate stress intensity factors at deepest point on the crack front and at a point near the surface.

For the surface flaw, the stresses normal to the plane of flaw at the flaw location are represented by a polynomial fit over the flaw depth derived using FEM is given by following relationship.

$$\sigma = A_0 + A_1 (x/a) + A_2 (x/a)^2 + A_3 (x/a)^3$$

Where  $A_0, A_1, A_2, A_3$  are the coefficients of stress distribution curve and x tends from 0 to a. Above equation provides accurate representation of stress over the flaw plane for all vessels of flaw depth having ratio  $x/a$  between 0 to 1.

The stress intensity factor using cubic polynomial stress distribution is given by

$$KI = [(A_0+A_p)]G_0+A_1G_1+A_2G_2+A_3G_3 \sqrt{\left(\frac{\pi a}{Q}\right)}$$

Where  $G_0, G_1, G_2, G_3 =$  Influence coefficients

$$Q = 1 + 4.593(a/l)^{1.65} - q_y$$

$q_y =$  Plastic zone correction factor

$$q_y = \{ [(A_0+A_p)]G_0+A_1G_1+A_2G_2+A_3G_3 / Sy \}^2 / 6$$

For fatigue crack growth  $q_y = 0$

Series of equations developed by J.M. Alegre and I.I. Cuesta [5] for determination of influence coefficients for various  $a/c$  and  $a/t$  ratios. The equations provide coefficients for both surface point and deep point on crack front. These equations are programmed using mathematical computation tool to give coefficients for all  $a/c$  and  $a/t$  ratios.

**FE Modeling and Analysis**

To evaluate SIF for semielliptical crack in case I and Case III fracture module of ANSYS Workbench 15.0 is used [6]. The crack is postulated by creating local coordinate system such as the maximum principal stress acts as a opening stress in mode I failure.

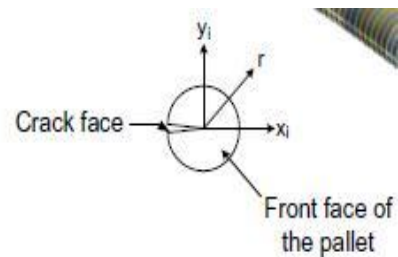


Figure 4: Orientation of Co-ord System at crack tip

For case I and case III sub modelling method is used for SIF evaluation by importing displacement field from uncracked body solution.

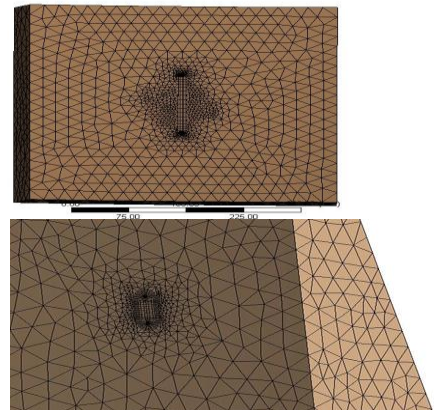


Figure 5: Sub model for Case I and case III

To determine SIF for quarter elliptical crack in nozzle shell junction no default module is available to postulate the crack. The quarter elliptical crack has been modelled separately and along the crack front torus body of 10 mm diameter is generated for use collapsible SOLID 186 -20 noded element.

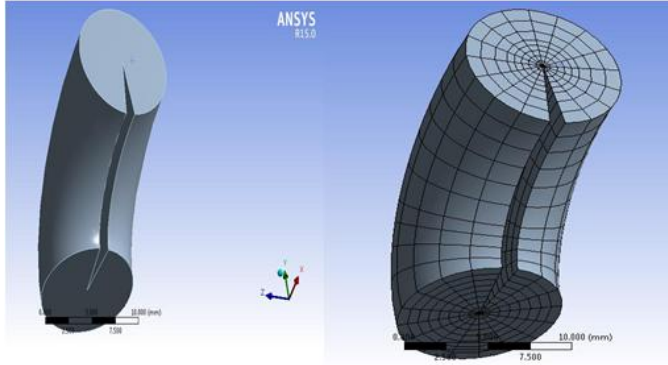


Figure 6: Meshed quarter elliptical crack body

Boundary conditions of pressure, thrust and displacement are applied on all three models and the simulation is solved for various a/c, a/t ratio and loading condition such as design and hydro test condition to check sensitivity of various parameters on SIF solutions.

**Determination of fracture toughness**

Plane strain condition fracture toughness value is used for analysis because of triaxial stress condition at crack tip. This provides conservative design and value of fracture toughness is constant and does not vary with thickness of plate. In this study fracture toughness is determined using impact energy given by Charpy- Izod impact test. The Charpy V –notch minimum energy value is taken from ARM document provided by supplier. This method is very economical and feasible. The relation between Charpy V-notch energy and fracture toughness is given in ASME SEC VIII, Div3 [4].

$$(KIC/Sy)^2 = 0.64(CVN/Sy-0.01)$$

Table 1: Fracture Toughness for various components.

Component	Yield Stress (Mpa)	Charpy Energy (J)	Fracture Toughness (Mpa√m)
Shell	27.6	27	76.91
Head	11.4	27	76.91
Nozzle	20.36	45	93.71

**Results and Discussion**

In present section, results are presented for all three cases considered (see Fig.2).

**Case I:**

The SIF distribution along the crack front for various a/c and a/t ratios at deep point is presented in Figs.7. The sensitivity analysis is performed using FEM for various a/c ratios as shown in Fig.8 and Fig.9

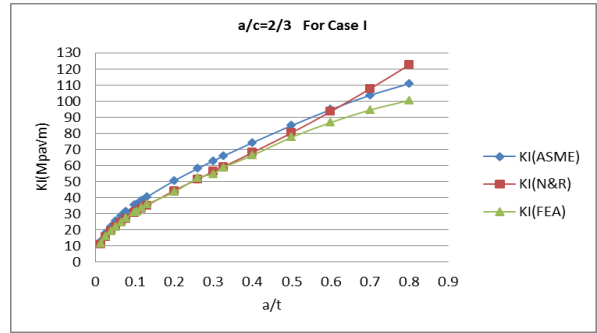


Figure 7: SIF for different a/t using various methods

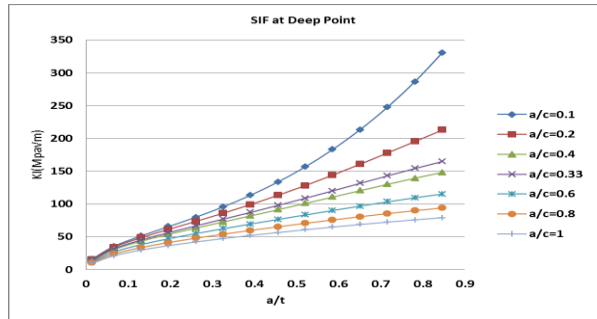


Figure 8: Sensitivity analysis at deep point

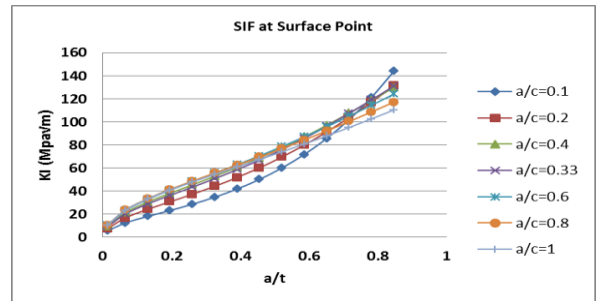


Figure 9: Sensitivity analysis at surface point

From Fig. 8 and Fig.9 it can be concluded that above a/c=0.5 the maximum SIF shifted from deep point to surface point of the crack front. The reason behind this may be considered as the influence of stress coefficient is increasing at surface point and the existence of stress tri-axiality.

**Case II:**

For quarter elliptical crack at nozzle shell junction the opening stress i.e. maximum principal stress magnitude is very high. The rate of increase of with increase in crack depth is highly sensitive as shown in Fig.10

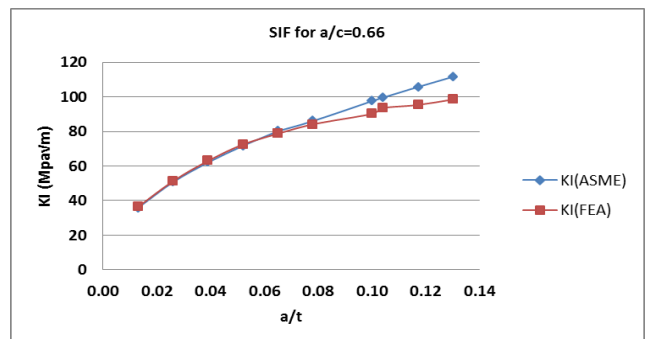


Figure 10: SIF for quarter elliptical crack at shell-nozzle junction

### Case III

This case refers to crack located at junction of shell with spherical head. There is thickness reduction from 76.8mm (shell thickness) to 41.8 mm (sphere thickness). The results given in Fig .11 refer to  $a/2c=1/3$  for  $a/t$  variation.

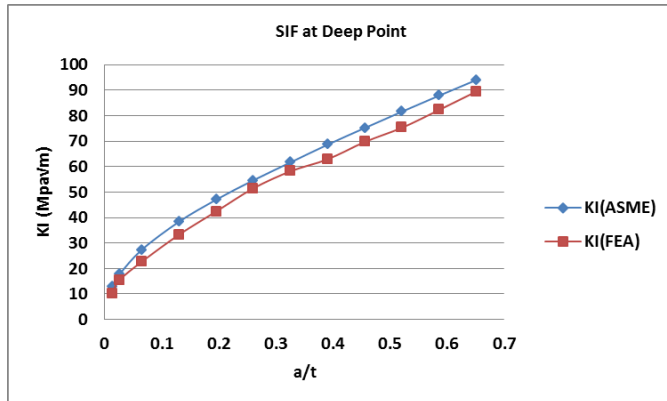


Figure 11: SIF for semi elliptical crack at shell-nozzle junction

### Conclusions

In the present study the SIF solutions of a surface semi elliptical and quarter elliptical crack located at different areas in pressure vessel have been derived using finite element methodology. The numerical solutions are verified using Newman & Raju SIF solutions and stress influence coefficient method from ASME. The use of stress influence coefficient method is extended for quarter elliptical crack by determining accurate stress distribution of opening stress by FEM. The main conclusion derived from above study can be summarized as follows:

- SIF results of crack located in areas away from geometrical discontinuity are less critical than crack located at discontinuity area.
- As  $a/c$  ratio goes beyond 0.5 the maximum SIF location on crack front shifted from deep point to surface point.
- The sub-modelling method used in ANSYS 15.0 is proven to be accurate and efficient for determination of SIF values. The reduction in computation time is very high as numbers of elements are reduced by great extent.
- The parametric FE modelling found to be highly efficient in sensitivity analysis of crack geometry in Case I
- The crack located at nozzle shell junction is more critical because of high stress concentration area.
- The use of pre-meshed crack module in ANSYS 15.0 allows modelling of various crack shape (quarter-elliptical) for determination of SIF.
- The extension of stress influence coefficient method given by ASME to quarter elliptical crack shows close agreement with results derived from FEM.
- The proposed methods may be extended to arbitrary geometrical configurations of pressure vessels and variable loading condition including thermal and residual stresses.

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