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Fatigue life and damage prediction of plate with central hole using finite element method

Vipin Wagare[1*] and Rashmi Hundekari*[2]

[1]*M.Tech.(Design),Dayanand Sagar College of Engineering, Bengaluru-560078,India.*
[2]*Sr.Scientist, CSIR-NAL,Bengaluru-560017,India.*

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Abstract

Estimation of fatigue life through testing is expensive and time consuming affair. In present fast pace product development senario,validated numerical simulations are considered to be one of the reliable source of preliminary fatigue life estimation. Numerical simulations may not prove to be a complete replacement to the fatigue testing but they can provide a detailed insight into the fatigue damage phenomenon. Present study demomstrates the finite element methodology adopted for accurate predcition of fatigue life and fatigue damage of a medium strength steel plate with hole at the centre. Crack initiation approach have been used for fatigue life estimation. Strain-life criterion is applied and number of cycles to crack initiation has been computed using Morrow's equation. Damage contours at the onset of crack intitation is plotted for various constant amplitude cyclic loads. Fatigue life predicted is in close agreement with the experimental results from literature. Maximum 3% of deviation has been observed when compared with experimental results. Finite element methdology demostrated in this work is further extended to evaluate fatigue life for the same structural element under variable amplitude loading. Cummulative fatigue life and damage under variable amplitude loading has been estimated using Miner's rule. Methodology used is generic in nature and can be used for estimation of fatigue life of real time structural components with complex geometries under a constant or variable amplitude loading.

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Introduction

Fatigue testing is a time consuming and expensive activity and most of the time it will be taken up in the last phase of product development. During fatigue testing if component fails to meet the specified intended life it needs to be redesigned and retested resulting in delay in the overall product development time. Keeping in view the amount of time and money involved in the fatigue testing; fatigue analysis through numerical simulation has been proved to be an effective method for fatigue life and damage prediction. Accurate fatigue life estimation plays a crucial role in ensuring structural integrity of the component throughout its intended operational life.

Present work demonstrates finite element methodology followed for predicting fatigue life of the structural element up to crack initiation and assessment of fatigue damage at the onset of crack. Crack initiation approach has been used for fatigue life and damage assessment. Estimation of fatigue life has been done based on Strain-life criterion. Morrow's equation has been used to calculate the fatigue life under constant amplitude cyclic loading. Fatigue life so estimated has been used to determine fatigue life under variable amplitude loading. Continuum damage law has been applied for predicting cumulative damage under variable amplitude loading.

Strain life approach for fatigue life estimation

Widely used strain based approach has been adopted for fatigue life estimation. From the experimental data available on fatigue it has been observed that the fatigue behaviour of a material can be accurately characterized by cyclic strain curves, plotted under constant amplitude, completely reversed straining with constant strain rate. Also it has been observed that the failure initiates at local plastic zones, crack nucleates and grows to a critical size due to plastic straining in localized zones. Hence a local strain based approach with a material model which captures cyclic stress strain behaviour has been selected for the present work. Elastic, plastic and cyclic stress-strain behaviour of the material has been captured using appropriate material model. Cyclic stress strain data available in literature [1] established using Romberg Osgood relationship has been used for cyclic strain computation. Surface finish effect has been accounted by including appropriate surface roughness values in to the fatigue model. Morrow's model which deals with mean stress effect has been used for accurate fatigue life prediction.

Cyclic stress strain computation

Stress strain behaviour of a material under inelastic cyclic reversals is different from strain obtained under monotonic elastic cyclic loading. Hence it is essential to capture cyclic stress stain behaviour to get accurate strain range and in turn accurate prediction of fatigue life using localized strain based method. Cyclic stress strain data compiled in literature [1] using Romberg Osgood relationship eq. (1) has been utilized in proposed study

$$\Delta\varepsilon_{eq} = \Delta\varepsilon_{eq}^{e} + \Delta\varepsilon_{eq}^{p} = \frac{\Delta\sigma_{eq}}{E} + 2\left(\frac{\Delta\sigma_{eq}}{2K'}\right)^{\frac{1}{n'}} \qquad (1)$$

Where,$\Delta\varepsilon_{eq}$ and $\Delta\sigma_{eq}$ are the equivalent range of local stress and strain; E is Young's Modulus; K' is cyclic hardening coefficient;n' is cyclic hardening exponent; and $\Delta\varepsilon_{eq}^{e}$ and $\Delta\varepsilon_{eq}^{p}$ are mean equivalent elastic and plastic strain range, respectively

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*Corresponding author. Tel./fax: +91 9742029639
E-mail address: hundekari.rashmi@gmail.com

Fatigue Model

Baseline strain life curve modified by Morrow to account for the effect of mean stress is chosen for carrying out the fatigue analysis using finite element analysis. Morrow altered the value of the fatigue strength coefficient in the elastic component of the stress-strain relationship for more accurate estimation. Morrow's fatigue model is expressed in eq. (2)

$$\frac{\Delta\varepsilon_{eq}}{2} = \frac{\sigma_f' - \sigma_m}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \qquad (2)$$

Where, $\Delta\varepsilon_{eq}$ is equivalent strain range, c is fatigue ductility exponent; ε_f' is fatigue ductility coefficient; b is fatigue strength exponent; σ_f' is fatigue strength coefficient and σ_m is local mean stress. Using Morrow's criterion fatigue life for various constant amplitude laoding have been determined.

Cumulative Damage Model

Fatigue Life estimated for constant amplitude loading have been further used to compute the fatigue life of same structural element under variable amplitude loading. Cumulative damage law established by M.A. Miner and known as Miner's Rule has been used to predict the fatigue life under variable amplitude cyclic loadings. Miner's rule accurately predicts the cumulative fatigue damage up to crack initiation phase due to slip band formations, micro cracks and dislocation. This law states that the damage fraction (D) at given constant stress level is equal to the number of applied cycles (n_i) at given stress level divided by the fatigue life (N_f) at that same stress level. It is expressed as in eq. (3)

$$D = \sum_{i=1}^{K} \frac{n_i}{N_f} \qquad (3)$$

Where, n_i is actual cycle count; N_f is cycle count till failure average no of cycles to failure; K is stress level; D is the fraction of life consumed by exposure to various load cycles

Fatigue analysis using finite element method

Fatigue analysis has been carried out in three phases using

1. Static stress analysis to determine max strain range under given cyclic loading.
2. Estimating the fatigue life.
3. Establishing damage contours.
4.

Static stress analysis to determine max strain range under given cyclic loading

Maximum stress value is obtained by carrying out static analysis using commercially available ABAQUS software. Region corresponding to maximum stress of where crack is likely to initiate has been identified through the stress contours. For carrying out the static stress analysis elasto-plastic material model has been used in order to capture the stresses for range of loadings. Maximum stress value so obtained has been used for finding the strain range with the help of Romberg-Osgood eq. (1)

Estimating the fatigue life

This is the second phase in the fatigue analysis. Strain based approach has been used for fatigue life estimation. Morrow's criterion which deals with the mean stress effect has been applied for accurate fatigue life estimation. Strain range results obtained from first phase using Romberg-Osgood equation has been used to estimate cycles to crack initiation.

Establishing fatigue damage contours

Cumulative fatigue damage has been calculated using a continuum damage model. In this damage model continuum damage occurred during individual load cycle has been summed up to calculate the total damage at the end of the fatigue cycles. This continuum model considers that the rate at which damage occurs is not linear, but is related to the damage already accumulated from the previous load cycles. An incremental damage procedure has been used to calculate the number of repetitions of the block loading up to crack initiation. An incremental damage procedure calculates the no of block loadings leading to 0.1 damage fraction. Subsequent to this damage parameters are modified as described in eq. (4) procedure has been repeated for each increment of 0.1 damage fraction till the Miners damage fraction become 1.AT the end of the analysis a damage contour has been established which further can used for crack growth analysis using appropriate progressive damage models.
The incremental fatigue damage is calculated using eq. (4)

$$\Delta D = \frac{(1-D_i)^{P_i}}{(P_i+1)N_{fi}} \qquad (4)$$

Where,
ΔD is the damage for the cycles in current damage increment
D_i is the damage so far accumulated
P_i is the damage rate parameter so far
N_{fi} is the endurance of cycle

P_i For a cycle is defined by the relationship in eq. (5)

$$P_i = 2.55\,(\sigma_{max}\,\varepsilon_a)^{-0.8} \qquad (5)$$

Fatigue analysis of a steel plate with hole at the centre

Fatigue life assessment is carried out for medium strength steel 100 mm long x 25.6 mm wide x 7.68 mm thick plate with a hole of diameter 12.8 mm at the centre. The plate geometry under consideration is shown in Fig.1

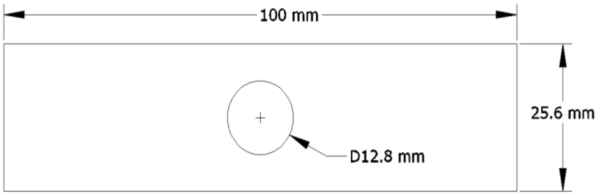


Figure 1: Geometrical details of specimen

Mechanical and cyclic properties of medium strength steel used during analysis have been tabulated in Table 1. Plate having a central hole of diameter 12.8 mm is subjected to uni-axial completely reversed cyclic loading i.e. stress ratio(R) = -1.

Table 1: Material properties for Medium strength steel

Parameter	**Notation**	**Values**
Static Properties		
Modulus of Elasticity (MPa)	E	206900
Poisson's ratio	ν	0.32
Yield Stress (MPa)	σ_y	648.3
Ultimate Stress (MPa)	σ_u	786.2
Cyclic Properties		
Fatigue Ductility coefficient	ε_f'	1.142
Fatigue Ductility exponent	c	-0.67
Fatigue Strength coefficient (MPa)	σ_f'	1165.6
Fatigue Strength exponent	b	-0.081
Cyclic strength coefficient (MPa)	k'	1062.1
Cyclic strain hardening exponent	n'	0.123

Finite element modelling

The specimen with hole at the centre is modelled using three dimensional deformable solid elements. Model has been meshed with C3D8R (8-node linear brick) elements available in ABAQUS software. The mesh size and mesh pattern has been finalised based on the convergence studies carried out before proceeding for the full analysis. Series of analysis have been carried out for various uni-axial constant amplitude cyclic loadings compiled at Table 2. Loads have been applied along length direction of the plate .FE Model and Mesh details of the specimen are as shown in Fig. 2 (a) & (b).

Table 2: Load data

S.N	Load (kN)
1.	62.25
2.	56.29
3.	53.89
4.	47.39
5.	40.18
6.	40.14
7.	31.14
8.	25.27
9.	22.02
10.	20.92

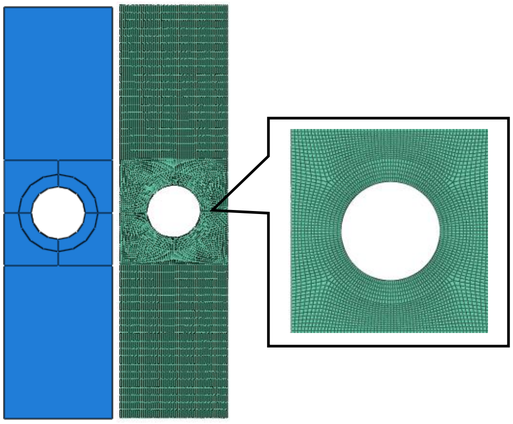

(a) (b)

Figure 2: (a) FE Model of Specimen (b) Mesh details near hole

Material modeling

Static stress analysis has been carried out using elasto-plastic material model as the load levels are ranging from linear elastic to plastic. As the loading is cyclic and it is important to capture the cyclic stress strain behaviour of the material for accurate strain based life prediction. Cyclic stress –strain data obtained in literature [1] using Romberg-Osgood equation (1) has been used to compute the strain range for given loading and same has been compiled at Table 3. The Morrow's fatigue model has been used for computing the fatigue life

Table 3: Cyclic stress strain data for medium strength steel[1]

No.	σ_a[MPa]	ε_a
1	0	0
2	50	2.42E-04
3	100	4.83E-04
4	150	7.25E-04
5	200	9.68E-04
6	250	1.22E-03
7	300	1.48E-03
8	350	1.81E-03
9	400	2.29E-03
10	450	3.10E-03
11	500	4.61E-03
12	550	7.41E-03
13	600	1.25E-02
14	650	2.16E-02
15	700	3.71E-02
16	750	6.28E-02
17	800	1.04E-01
18	850	1.68E-01

Results and Discussion

Static Stress analysis results

For the constant amplitude loads givenin Table2series of static stress analysis have been carried out and the maximum stress values have been extracted through commercially available FEM software. Maximum stress levels so obtained are compared against the stress values obtained in literature [1]. A specimen stress contour for 31.14kN load has been shown in Fig.3.For other load cases maximum stress value obtained from FEM are compiled at Table 4.

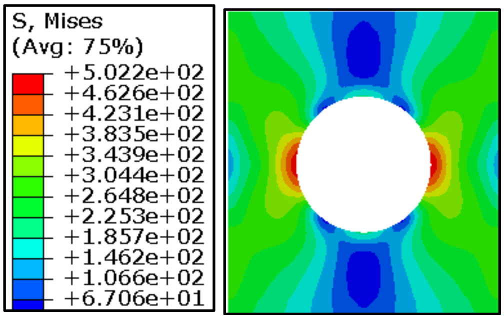


Figure 3: Stress contours for P =31.14kN

Fatigue life predictions for constant amplitude loading

Number of cycles to crack initiation obtained through fatigue analysis and its comparison against existing experimental results from literature [1] has been presented in Table 4.The fatigue damage contours for all the load cases depicting the likely location of the crack initiation in the vicinity of hole has been given in Table 7. Red zone in the damage contour indicates the crack initiation location. Information related to crack initiation location can be further used to carry out the crack growth analysis.

Table 4: Number of cycles to crack initiation

Load (kN)	Max. Vonmises Stress (MPa) Literature [1]	Max. Vonmises Stress (MPa) FEM	Fatigue life N_f (cycles) By Experiment[1]	Fatigue life N_f (cycles) By FEM
62.25	722.50	736.7	68	66
56.29	671.90	681.4	190	195
53.89	653.60	661.8	265	258
47.39	602.10	612.6	1250	1224
40.18	550.30	563.9	2400	2389
40.14	550.10	563.7	3600	3623
31.14	485.90	502	11500	11375
25.27	439.40	448.7	55400	56710
22.02	407.00	409.2	160780	163692
20.92	394.60	394.8	188000	192177

Fatigue life estimation for variable amplitude loading

Real time service loads are always of variable amplitude loads pertaining to various service conditions. To demonstrate FEM methodology applied for fatigue life estimation under variable amplitude loading same plate with central loading has been considered for fatigue analysis. Accurate representation of variable amplitude lading is important for accurate fatigue life predictions. The load spectrum shown in Fig: 4 taken from literature [1] have been used for life estimation and have been represented as a single block load using tabular cyclic load input option available in ABAQUS.

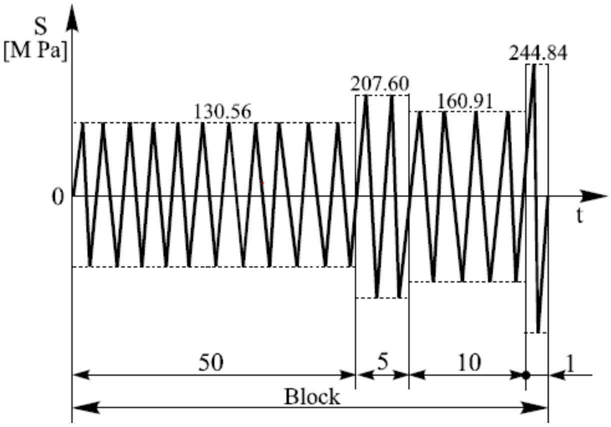


Figure 4: Load spectra

Linear static stress analysis for given block loading has been carried out using FEM. Strain range corresponding to maximum stress has been obtained from cyclic stress strain data.

Using Morrow's fatigue model number of block repetitions up to crack initiation has been predicted. In addition to FEM fatigue life under variable amplitude has also been computed analytically using cumulative damage law. Fatigue life obtained for constant amplitude loading has been used to compute the individual damage fractions and Palmgren-Miner's rule has been applied to obtain cumulative damage. Number of blocks of loading up to crack initiation and the damage contours under given load spectra has been given in Table 5 and Table 6.

Table 5: No. of cycles to crack initiation under variable amplitude loading

Ni	Load (P)	Max stress MPa	N_f (cycles)		N_{bl} (cycles)	
			Experimental	FEM	Analytical	FEM
50	25.27	130.56	55400	56710	214	209
5	40.18	207.60	2400	2389		
10	31.14	160.91	11500	11375		
1	47.39	244.84	1250	1224		

Table 6: Number of cycles to crack initiation for variable amplitude loading shown in Fig: 4

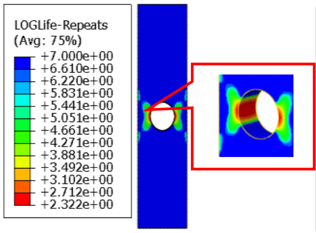
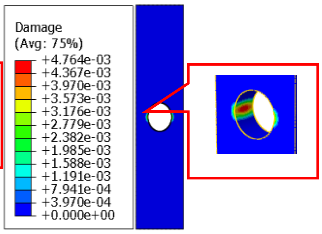
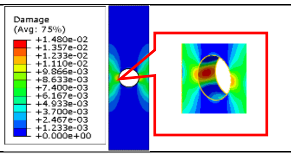
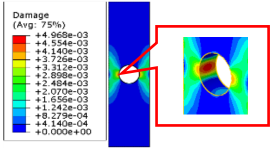
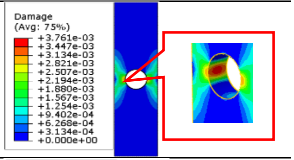
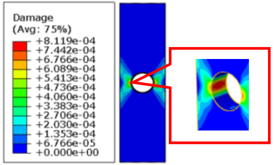
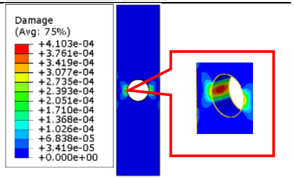
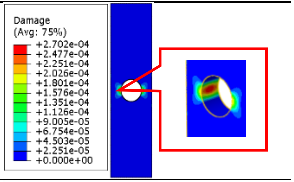
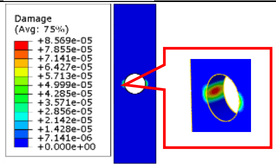
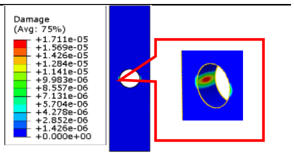
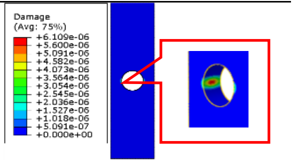
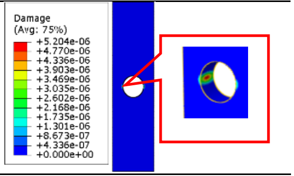
Life contours	Damage contours
	

Table 7: Damage contours

S.N	Load (kN)	N_f (Cycles to crack initiation)	Damage contours
1	62.25	66	
2	56.29	195	
3	53.89	258	
4	47.9	1224	
5	40.18	2389	
6	40.14	3623	
7	31.14	11375	
8	25.27	56710	
9	22.02	163692	
10.	20.92	192177	

Conclusions

Present work demonstrates the finite element methodology to be adopted for carrying out fatigue analysis. Fatigue analysis has been carried out for a standard specimen of a medium strength steel plate with hole at the centre subjected to cyclic loading. Cyclic stress strain behaviour, surface roughness and mean stress effects have been accounted in the while estimating the fatigue life and predicting the fatigue damage. For realistic representation of variable amplitude loading; loads have been represented using tabulated cyclic load input. Predictions obtained from the fatigue analysis carried out using finite element method for constant amplitude and variable amplitude loading show close agreement with the experimental results. Deviation observed from experimental results is within 3%. Thus the methodology has been verified. Damage contours obtained give useful information regarding the crack initiation location and its orientation which can be further used for carrying out crack growth analysis or progressive damage and failure prediction. Methodology is generic in nature and can be extended to fatigue life estimations of structural elements with complex geometry multi axial loading.

Acknowledgement

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