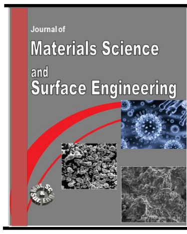
Journal of Materials Science & Surface Engineering Vol. 3 (3), 2015, pp 257-261




Contents lists available at http://www.jmsse.org/

Journal of Materials Science & Surface Engineering

# EPFM Analysis of a Steam Generator Tube with an Axial Crack: Nonlinear Energy Release Rate (J) along the Crack Front

Sumandas D.[1*] and H.V.Lakshminarayana[2]

[1]*Department of Mechanical Engineering, P.E.S. College of Engineering-571401, Mandya.*
[2]*Department of Mechanical Engineering, Dayananda Sagar College of Engineering-560078, Bengaluru.*

Article history

Received: 05-June-2015
Revised: 15-June-2015
Available online: 04-Sep-2015

**Keywords:**
Elastic-plastic fracture mechanics,
Nonlinear energy release rate (J),
Steam generator tube,
Plastic singular elements,
Finite Element analysis.

## Abstract

Fracture is a failure mode due to unstable crack propagation under applied stress. Fracture mechanics provides a methodology for prediction, prevention and control of fracture in materials, components and structures. Elastic-plastic fracture mechanics applies to materials that exhibit time-independent, nonlinear behaviour (i.e., plastic deformation). Nonlinear energy release rate (J) describes a crack tip conditions in elastic-plastic materials and can be used as a fracture criterion. Critical values of J give nearly size independent measures of fracture toughness, even for relatively large amount of crack tip plasticity. Elastic-plastic fracture is computed using a plastic singular element around the crack tip. The finite element model developed using ABAQUS is validated using NAFEMS benchmark namely centre cracked rectangular panel under remote tensile stress for which target solutions are available. The use of computed J is demonstrated to investigate failure of nuclear reactor steam generator tube with an axial through thickness crack subjected to internal pressure. The variations of J along the crack front are computed. Parametric studies are carried out to quantify the effect of crack length, material models [elastic perfectly plastic, bilinear [E, $E_T$], Ramberg Osgood law (n)] on J.

*The work had been presented at an international conference **Fatigue Durability India 2015**, 28-30th May 2015, JN TATA AUDITORIUM, Indian Institute of Science, Bangalore.* 

## Introduction

Fracture is a failure mode due to unstable crack propagation under applied stress. Fracture mechanics provides a methodology for prediction, prevention, and control of fracture in materials, components and structures subjected to static, dynamic, and sustained loads.

Linear elastic fracture mechanics is valid only as long as nonlinear material deformation is confined to a small region surrounding the crack tip. In many materials it is impossible to characterize fracture behaviour with LEFM. Limited plasticity at the crack tip was accounted for through plasticity correction. The approach is more than sufficient for a wide variety of problems, notably Fatigue Crack Growth(FCG) prediction for which the maximum stress due to applied loads are less than 30 percent of the yield stress. When ductile metals are loaded beyond elastic range, the initial linear stress response will give way to a complicated nonlinear response. Therefore LEFM is abounded and EPFM is embraced for intended analysis of computational fracture mechanics.

*Elastic Plastic Fracture Mechanics:*

Elastic plastic fracture mechanics applies to material that exhibit time-independent, nonlinear behaviour (i.e., plastic deformation). In EPFM there are two parameters CTOD and nonlinear energy release rate (J). Both these parameter describe crack tip conditions in elastic plastic materials, and each can be used as a fracture criterion. Critical values of CTOD or J give nearly size independent measure of fracture toughness, even for relatively large amounts of crack tip plasticity. There are limits to the applicability of J and CTOD, but these limits are much less restrictive than the validity requirements of LEFM.

*Nonlinear Energy Release Rate (J):*

Energy release rate in a nonlinear elastic body that contains crack is known as nonlinear energy release rate (J). For 2D plane problems the J is evaluated as a contour integral as shown in Fig.1.

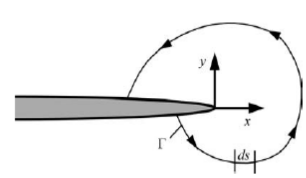



**Figure 1:** Arbitrary Contour Around the Tip of a Crack.

$$J = \int_{\Gamma} \left( w\,dy - T_i \frac{\partial u_i}{\partial x} ds \right)$$

………(1)

where,

w= strain energy density
$T_i$= components of the traction vector

Corresponding author. Tel./fax: +919980323690
E-mail address: sumandas625@gmail.com

$u_i$= displacement vector components
ds= length increment along the contour Γ

The strain energy density is defined as

$$w = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \quad \text{……..(2)}$$

where, $\sigma_{ij}$ and $\varepsilon_{ij}$ are the stress and strain tensors, respectively. The traction is a stress vector at a given point on the contour. That is, if we were to construct a free body diagram of the material inside of the contour, *Ti* would define the stresses acting at the boundaries. The components of the traction vector are given by,

$$T_i = \sigma_{ij} n_j \quad \text{……(3)}$$

where, $n_j$ are the components of the unit vector normal to Γ. Rice showed that the value of the *J* integral is independent of the path of integration around the crack. Thus *J* is called a path-independent integral.

*Computational Elastic-plastic Fracture Mechanics:*

Most FEA applications undertaken by design engineers were limited to linear analysis. Such linear analysis provides an acceptable approximation of real-life characteristics for most problems design engineers encounter. Nevertheless, occasionally more challenging problems arise, problems that call for a nonlinear approach. There are three types of non linearity: material non linearity, geometric non linearity and contact non linearity. The present analysis mainly focuses on material non linearity. In elastic problems, the nodes at the crack tip are normally tied, and the mid-side nodes moved to the ¼ points as shown in Fig.2a such a modification results in a 1/√r strain singularity in the element, which enhances numerical accuracy. When a plastic zone forms, the 1/√r singularity no longer exist at the crack tip. Consequently, elastic singular elements are not appropriate for elastic-plastic analyses. Fig. 2b shows an element that exhibits the desired strain singularity under fully plastic conditions.

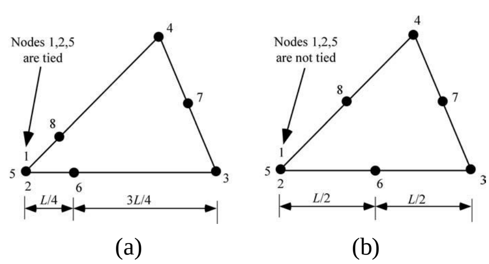



**Figure 2:** Crack-tip Elements for Elastic and Elastic-Plastic Analyses. Element (a) Produces a 1/√r elastic Strain Singularity, while (b) Exhibits a 1/r plastic Strain Singularity

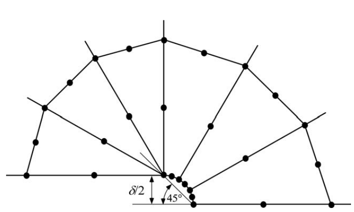



**Figure 3:** Deformed Shape of Plastic Singularity Elements.

Note that three nodes occupy the same point in space. Fig. 5 shows the analogous situation for three dimensions, where a 20-noded hexahedral Solid element is degenerated into a 15-noded wedge element.

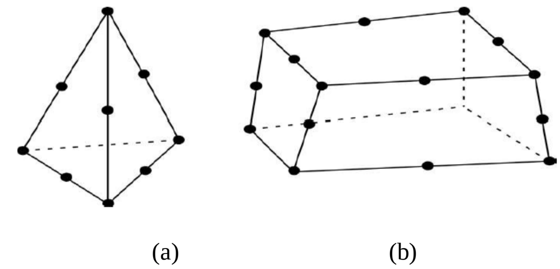



**Figure 4:** Common Three-Dimensional Continuum Finite Elements: (a) Tetrahedral Element and (b) Brick Element

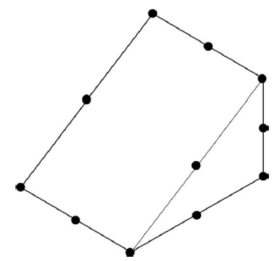


**Figure 5:** Degeneration of a Brick Element into a Wedge (SPENTA 15)

## Finite Element Model Development

A standard test problem with known target solutions in the form of formulae, graphs, tables obtained using analytical methods, experimental techniques, and computational procedures. Used to validate finite element modelling for engineering analysis of candidate components and structures using a chosen commercial FEA software ABAQUS.

*Benchmark: Centre Cracked Rectangular Panel under Remote Tensile Stress*

A centre cracked rectangular panel of width 2W = 100 mm, length 4W = 200 mm, crack length 2a = 20 mm, (a/w = 0.2) as shown in Fig.6. The material properties of nuclear alloy are Elastic modulus E=2e5MPa, poisons ratio ν=0.3and yield stress $\sigma_y$=271MPa as shown in Fig.7. A plane strain state is assumed and the material is elastic perfectly plastic.

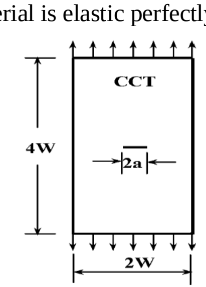



**Figure 6:** Centre Cracked Rectangular Panel Subjected to Remote Tensile Stress

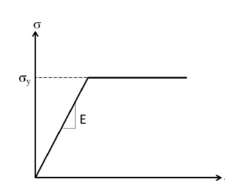



**Figure 7:** Stress-Strain Curve for Elastic Perfectly Plastic Material

*Finite Element Model:*

The typical finite element model is presented in Fig.8 and a refined mesh of STRIA 6 elements is generated near the crack tip, and a compatible mesh of QUAD8 elements is used in the rest of the domain.

Element type: CPE8R (An 8-node bi-quadratic plane strain quadrilateral, reduced integration.) Number of elements is 7854 and nodes are 39820.
Number of elements around the crack-tip = 72

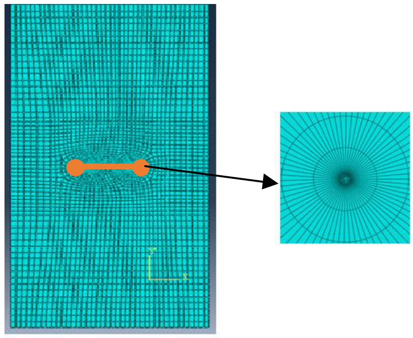


**Figure 8:** Finite Element Model and Singularity Elements Around the crack-tip.

*Validation:*

**Table 1:** J Value for Each Load Step

| Sl. No. | Load factor $\sigma/\sigma_Y$ | J $J/mm^2$ | Normalized J $EJ/(a*\sigma_Y^2)$ |
|---|---|---|---|
| 1 | 0.1 | 0.1842 | 0.0250 |
| 2 | 0.2 | 0.7372 | 0.1004 |
| 3 | 0.3 | 1.9532 | 0.2660 |
| 4 | 0.4 | 3.3725 | 0.4593 |
| 5 | 0.5 | 5.2245 | 0.7115 |
| 6 | 0.6 | 7.3426 | 0.1006 |
| 7 | 0.7 | 10.0901 | 1.3742 |
| 8 | 0.8 | 13.9713 | 1.9028 |
| 9 | 0.9 | 21.6946 | 2.9548 |
| 10 | 0.925 | 26.3458 | 3.5882 |

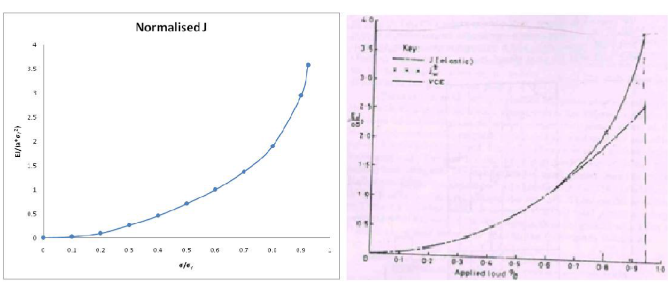



**Figure 9**: Graph of Normalized J v/s Load Factor Using FEM and Graph of Normalized J v/s Load Factor (NAFEM's Manual)

For each load step, the J is evaluated around the crack-tip and tabulated in table 1. The Normalized J curve is presented in Fig.9 and compared with the master curve in the NAFEMS document [1]. These results are found to closely match with the target solutions reported.

***Case Study***

A nuclear reactor steam generator tube with an axial through thickness crack subjected to an internal pressure as shown in a Fig 10. The material model considered is Elastic-Perfectly Plastic, Bi-linear material and Ramberg Osgood material. The objective is to perform elastic-plastic fracture mechanics analysis to compute Nonlinear Energy Release Rate (J) and its variation along the crack front.

The geometric modelling of a cylindrical tube with an axial through thickness crack is done using ABAQUS 10.0. The geometric parameters used in the computation are inner radius = 9.84mm, outer radius = 11.11mm, wall thickness t = 1.27mm, length 2L = 100mm, crack length 2a = 20mm. The geometric model is shown in Fig. 11.

The cylindrical model is meshed suitably using Hex20 element (C3D20R) in ABAQUS 10.0 as shown in Fig. 12. The total number of elements is equal to 20032 and nodes are 90951. A total of 72 singular elements are generated around the crack-tip, thus maintaining the singularity element angle of $5^0$ as shown in Fig. 12. The length of the singularity element '$\Delta a$' is equal to (a/100). The ends of the tube are constrained using rigid links. These rigid link elements enforce kinematic relationships between the displacements at two or more nodes in the analysis. The tube is subjected to pure internal pressure based on the material properties as shown in Fig. 13.

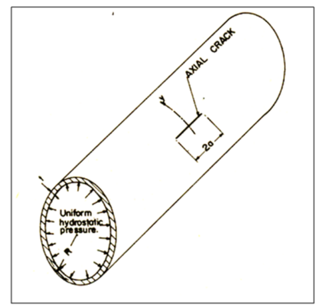



**Figure 10:** Cylindrical tube with a axial through thickness crack subjected to internal pressure.

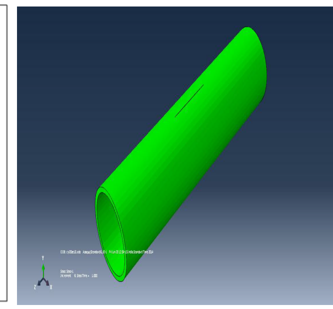


**Figure 11:** Geometric model of a cylindrical tube with axial through thickness crack.

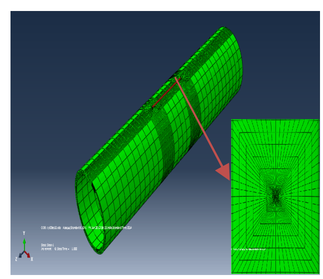


**Figure 12:** Finite element model of a cylindrical tube.

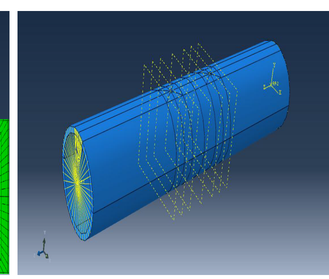


**Figure 13**: Boundary condition and loading.

*Graphical post-processing:*

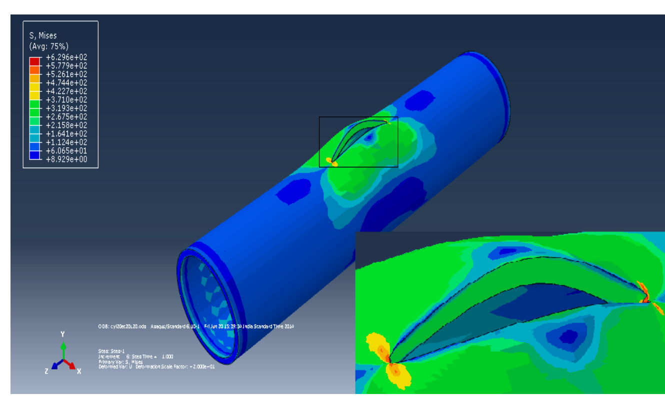



**Figure 14:** Distribution of von-Mises Stress Contours across the Pressurized Tube.

The von Mises stresses of the cylindrical tube are presented in Fig. 14 and Fig. 15 shows a crack-tip blunting on the outer and the inner surface of the tube. The von Mises stress developed here is 629.6 N/mm2 and it is more than the yield value. Therefore, onset of yielding will takes place and plastic zone is created near the crack-tip as shown in the Fig. 14.

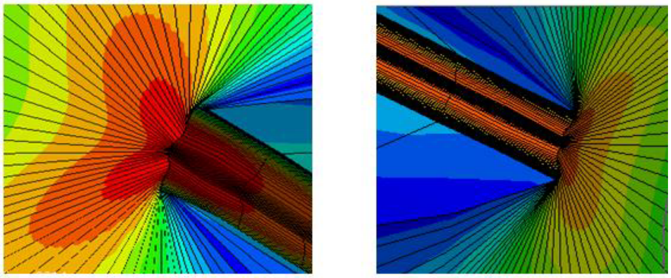


**Figure 15:** Crack-Tip Blunting on the Outer and Inner Surface of the Tube.

*Parametric Study:*

Parametric study has been done for two crack lengths (20 and 30), material properties (elastic-perfectly plastic, bi-linear and Ramberg Osgood). The results are presented in graphs.

*Case 1: Effect of Crack Length for Elastic Perfectly Plastic Material*

Hoop Stress, $\sigma = (pr)/t$ ……… (4)

where,

σ is the hoop stress.

P is the applied pressure.

r is the inner radius.

t is the thickness.

Therefore, P corresponds to applied pressure and $P_{max}$ corresponds to Yield stress of the material.

The cylindrical tube with axial crack is analyzed with varying crack length and material properties. The J and its normalized values are evaluated along the crack front and are presented below

**Table 2**: Normalized J vs Crack Front Variation for Elastic Perfectly Plastic Material

| Crack front (mm) | J ($J/mm^2$) a:l=0.2 $P/P_{max}$=0.2285 | J ($J/mm^2$) a:l=0.3 $P/P_{max}$=0.1715 | Normalized J $EJ/(a*\sigma_Y^2)$ a:l=0.2 | Normalized J $EJ/(a*\sigma_Y^2)$ a:l=0.3 |
|---|---|---|---|---|
| 0.0 | 0.9635 | 1.2147 | 0.1312 | 0.1103 |
| 0.15875 | 5.2246 | 9.2008 | 0.7116 | 0.8354 |
| 0.3175 | 7.5338 | 12.1814 | 1.0261 | 1.060 |
| 0.47625 | 9.5356 | 15.7162 | 1.2987 | 1.3779 |
| 0.635 | 11.4468 | 18.8327 | 1.5590 | 1.7100 |
| 0.79375 | 12.1546 | 19.6834 | 1.6554 | 1.7872 |
| 0.9525 | 10.7617 | 17.1507 | 1.4657 | 1.5573 |
| 1.11125 | 7.7178 | 11.4738 | 1.0511 | 1.0418 |
| 1.27 | 0.6012 | 1.1332 | 0.0818 | 0.1028 |

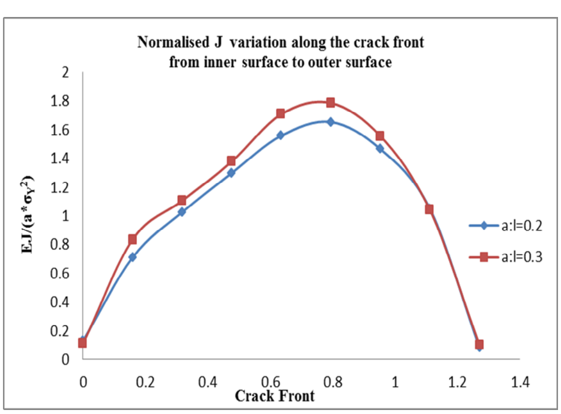



**Figure 16:** Normalized J vs Crack Front Variation for Elastic Perfectly Plastic Material

*Case 2: Effect of crack length for Bi-linear material*

To study the effect of Tangent modulus on the behaviour of the cylindrical tube with an axial through thickness crack subjected to internal pressure, computations are performed with tangent modulus of 10,000 $N/mm^2$ and 20,000 $N/mm^2$. The geometric modelling, element type, mesh option, boundary condition, solution control and post-processing option remain the same as the above case. As crack length increases the maximum load decreases and corresponding results are presented in graph.

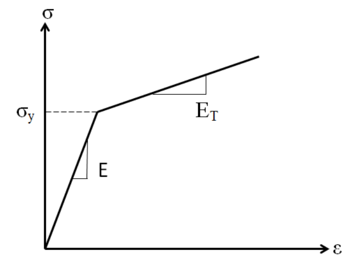



**Figure 17:** Stress-Strain Curve for Bi-linear material

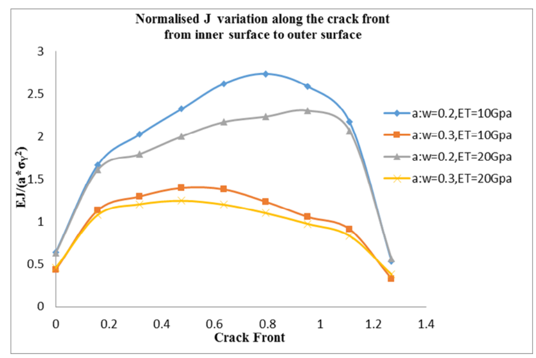



**Figure 18:** Normalized J vs Crack Front Variation for Tangent Modulus $E_T$= 10,000 and 20,000 $N/mm^2$

*Case 3: Effect of Crack Length for Ramberg Osgood Material*

To study the effect of strain hardening exponent on the behaviour of the cylindrical tube with an axial through thickness crack subjected to internal pressure, computations are performed with strain hardening exponent n is 3 and 5. The geometric modelling, element type, mesh option, boundary condition, solution control and post-processing option remain the same as the above case. As crack length increases the maximum load decreases and corresponding results are presented in graph.

**Table 3:** Ramberg Osgood Material Property for Nuclear Alloy (600) Steel

| Material model | E MPa | ν | $\sigma_y$ MPa | $\sigma_{ult}$ MPa | $\varepsilon_y$ | n | α |
|---|---|---|---|---|---|---|---|
| Ramberg Osgood law | 2e5 | 0.3 | 271 | 634 | 0.00135 | 3&5 | 0.02 |

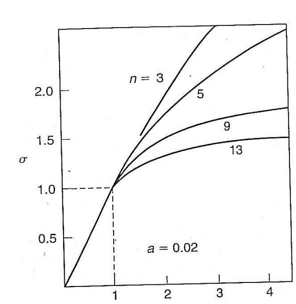



**Figure 19:** Stress-Strain Curve for Ramberg Osgood Material

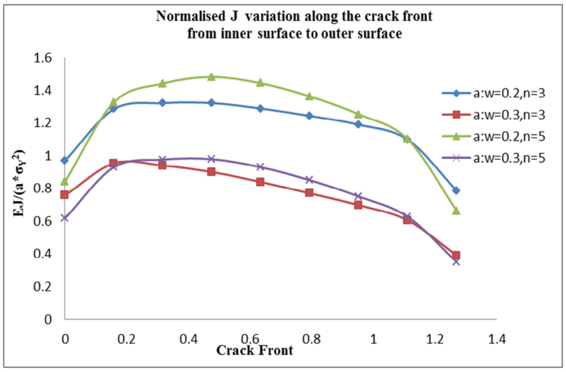



**Figure 20:** Normalized J vs Crack Front Variation for Strain Hardening Exponent n=3 and n=5.

## Conclusions

1. Finite element modeling using ABAQUS software for J evaluation in general and its variation along a crack front for a pressurized tube with an axial through the thickness crack is demonstrated in this investigation.
2. Ductile fracture handbook provides J solution for a large number of 2D cracked body problems. However, the variation of J along the crack front is not reported.
3. Ductile fracture handbook does not provide J and its variation along a crack front for pressurized tube with an axial through crack problem. This gap is hopefully bridged in this study.
4. The use of Isoparametric elements is well established in computational LEFM. A modest effort in this direction for computational EPFM is the use of plastic singularity elements.
5. J evaluation is the first step in EPFM analysis of a pressurized tube with an axial through crack. Prediction of residual strength and remaining life deserves an in-depth study of fracture criteria and fatigue crack growth laws.
6. The above predictions have to be verified using fatigue and fracture test results. Experimental investigations are indispensible.

## References

1. E. Hinton (Editor) *NAFEMS Introduction to Non Linear Finite Element Analysis*, NAFEMS, Glasgow, UK 1992.
2. J.H. Lee , Y.W. Park , H.D. Chung , Y.H. Choi , S. Majumdar, *Investigation on Behavior of Trapezoidal Through-Wall Cracks Under Constant Pressure Loading*, International Journal of Pressure Vessels and Piping, 85,2002, 606– 611.
3. Yun-Jae Kim, Nam-Su Huh, Young-Jae Park, Young-Jin Kim, *Elastic-Plastic J and COD Estimates for Axial Through-Wall Cracked Pipes*, International Journal of Pressure Vessel and Piping, 79, 2002, 451-464.
4. Jacob Foxen and Sharif Rahman, *Elastic-Plastic Analysis of Small Cracks in Tubes Under Internal Pressure and Bending*, Nuclear Engineering and Design, 197, 2000, 75-87.
5. Robert Firmature and Sharif Rahman, *Elastic-Plastic Analysis of Off-Center Cracks in Cylindrical Structures*, Engineering Fracture Mechanics, 66, 2000, 15-39.
6. Sanjeev Saxena and D. S. Ramachandra Murthy, *On the Accuracy of Ductile Fracture Assessment of Through-Wall Cracked Pipe*, Engineering Structures, 29, 2007, 789-801.
7. Zahoor A., Ductile Fracture Handbook, *Axial Through Wall Crack*, Electric Power Research Institute Report NP-6301-D, 2, 1989, 580-589.
8. ABAQUS 6.10, GUI Toolkit User"s Manual Volume I: A-H, Dassault Systemes, France, 2011.
9. R. J. Sanford, *Principles of Fracture Mechanics*, Prentice Hall, Pearson Education Inc., Upper Saddle River, New Jersey, 1st ed., 2003.